into (5.213) we obtain exactly the inverse of the matrix $\boldsymbol{K}_c$

$$\boldsymbol{K}_c^{-1} = \begin{bmatrix} 0.78 & 0.56 & 0.33 & 0.22 \\ 0.56 & 1.11 & 0.67 & 0.44 \\ 0.33 & 0.67 & 1.0 & 0.67 \\ 0.22 & 0.44 & 0.67 & 0.78 \end{bmatrix} = \boldsymbol{K}^{-1} + \boldsymbol{K}^{-1}\boldsymbol{U}(\boldsymbol{I} - \boldsymbol{V}^T\boldsymbol{K}^{-1}\boldsymbol{U})^{-1}\boldsymbol{V}^T\boldsymbol{K}^{-1}\,. \tag{5.217}$$

A comparison with the original $\boldsymbol{K}^{-1}$ confirms: Stiffness changes affect ***all*** influence functions, because no column of $\boldsymbol{K}^{-1}$ made it to $\boldsymbol{K}_c^{-1}$.

5.24 One-Click Reanalysis

In the program BE-FRAMES, see Chap. 10, both techniques to calculate $\boldsymbol{u}_c$, iteration and direct solution, are implemented as *One-Click Reanalysis*. The student can make changes to a frame by clicking on single elements and she or he immediately sees the effects, as in Fig. 5.32.

A click on an element means a change in the element matrix of the type $\boldsymbol{\Delta K}_e = c \cdot \boldsymbol{K}_e$ where c is a preset scaling factor; $c = 0$ would correspond to a complete loss of the element.

A series of clicks on, say, the elements 5, 7 and 9, simply means that $\boldsymbol{\Delta K}$ is a collection of scaled element matrices $\boldsymbol{\Delta K}_e$

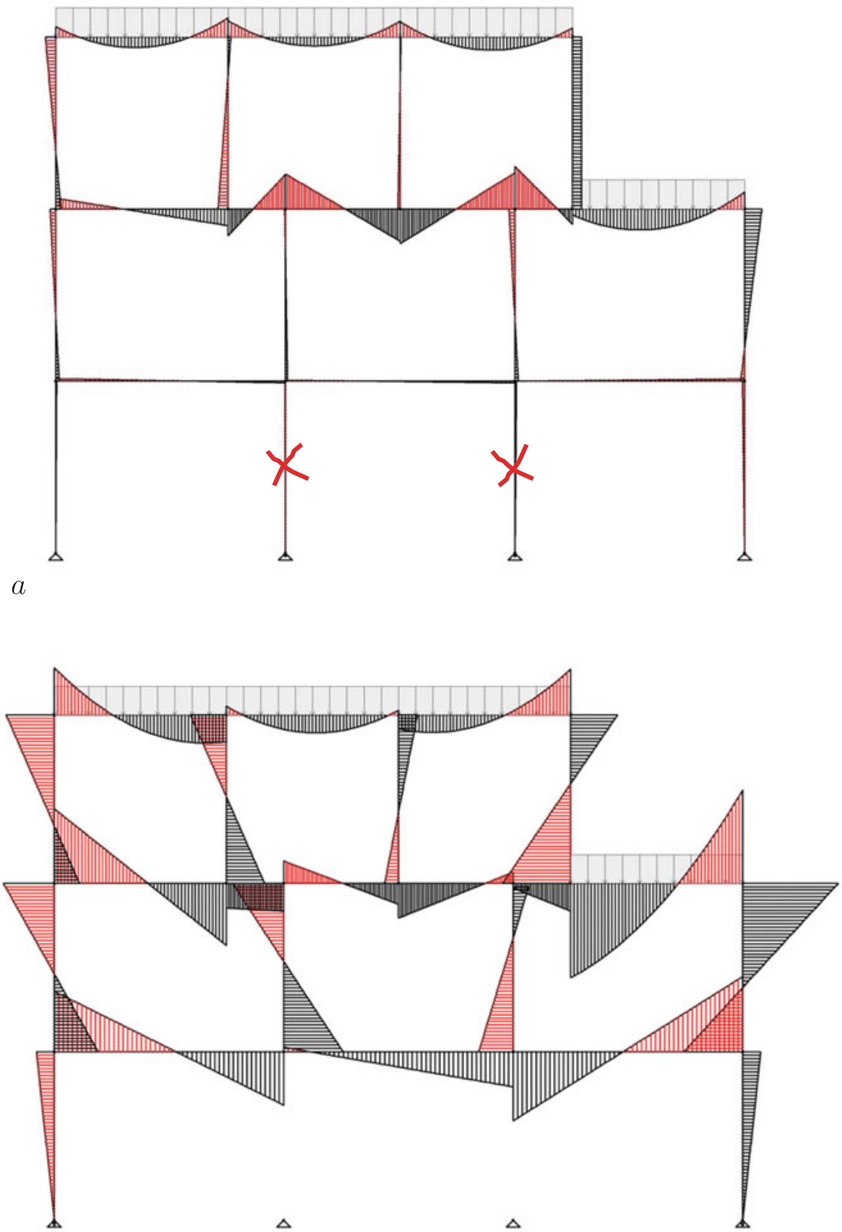
$$\boldsymbol{u}_c = -\boldsymbol{K}^{-1}\,(\boldsymbol{\Delta K}_5 + \boldsymbol{\Delta K}_7 + \boldsymbol{\Delta K}_9)\boldsymbol{u}_c + \boldsymbol{u} \qquad \text{(direct sol.)} \tag{5.218}$$

5.24.1 *When the Load “Is Hit”*

If the student removes an element Ω_e, which carries a load, the new vector $\boldsymbol{u}_c$ is the solution of the system

$$(\boldsymbol{K} + \boldsymbol{\Delta K})\boldsymbol{u}_c = \boldsymbol{f} - \boldsymbol{f}_e\,, \tag{5.219}$$

where the entries in the vector $\boldsymbol{f}_e$ are the previous equivalent nodal forces of the dropped element Ω_e. In this case the program must also change the original right side, $\boldsymbol{f} \to \boldsymbol{f} - \boldsymbol{f}_e$.



a

b

Fig. 5.32 One-click reanalysis (without modification of the stiffness matrix), **a** the crossed-out elements, **b** moment distribution without the two columns

5.24.2 Singular Stiffness Matrices

Since it is implemented in the program BE-FRAMES, we mention that the Gauss algorithm can also detect possible rigid body motions in frames. The Gauss algorithm transforms the stiffness matrix into an upper triangular matrix. If the algorithm is applied to the upper triangular matrix as well, the result is the unit matrix—for regular matrices. However, if the matrix is singular, the last columns are the eigenvectors to the eigenvalue(s) $\lambda = 0$, which can be displayed on the screen to notify the user of missing restraints, see Fig. 5.33.

The matrix created by this "twofold" Gauss is called the ***row reduced echelon form*** of a matrix and under this name the algorithm can be found in the literature.

5.25 Subsequent Installation of Joints

The student can also add joints to a frame and study how this affects the results. Mathematically a joint is a zero in $M(x) = 0$. If the student clicks at a point x_0 a Dirac delta δ_2 is applied at this point, the influence function for $M(x_0)$ is calculated and the moment $M_2(x_0)$ of the influence function itself at this point. Then the Dirac delta is scaled by a factor a to achieve $a \cdot M_2(x_0) + M(x_0) = 0$ and the moment distribution in the frame + hinge becomes $M_c(x) = M(x) + a \cdot M_2(x)$, see Fig. 5.34.

When multiple joints are added, a linear system controls the mutual influence of the different Dirac deltas.

5.26 Buckling Loads

The buckling loads of a frame are the eigenvalues of the second-order stiffness matrix $\boldsymbol{K}$. These too will be affected by modifications in the stiffness, $EI \rightarrow EI + \Delta EI$.

Here we only study the simplest case and assume the system to depend on a parameter t and therewith also the eigenvalues $\lambda(t)$ and the eigenvectors $\boldsymbol{u}(t)$

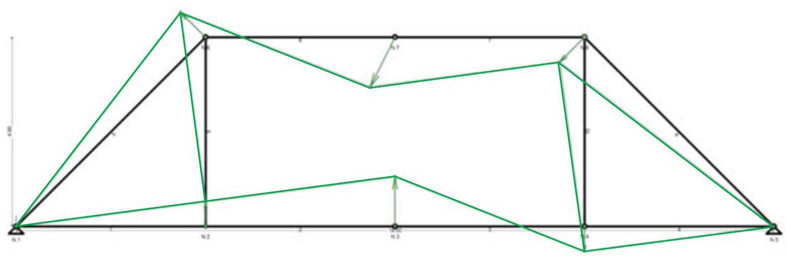


Fig. 5.33 Possible infinitesimal motions in a frame can be discovered with the "twofold" Gauss

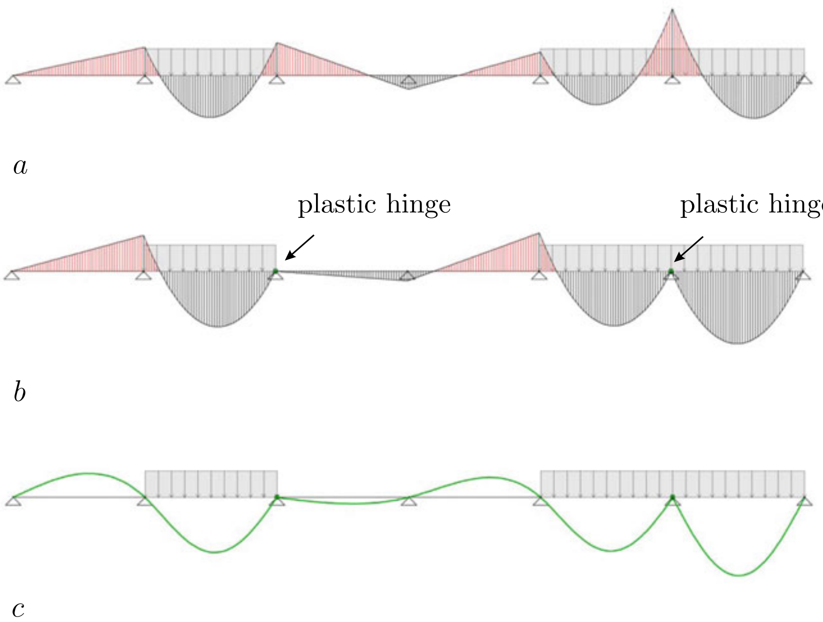


Fig. 5.34 Adding plastic hinges to a continuous beam, **a** original moment distribution, **b** after the hinges have been added, **c** deflection

$$\boldsymbol{K}(\lambda)\,\boldsymbol{u}(t) = \lambda(t)\,\boldsymbol{u}(t)\,. \tag{5.220}$$

The transposed matrix $\boldsymbol{K}^T$ has the same eigenvalues, but not necessarily the same eigenvectors $\boldsymbol{v}(t)$

$$\boldsymbol{K}^T(t)\boldsymbol{v}(t) = \lambda(t)\,\boldsymbol{v}(t)\,. \tag{5.221}$$

One can always normalize the eigenvectors of $\boldsymbol{K}$ and $\boldsymbol{K}^T$ belonging to $\lambda(t)$, so that $\boldsymbol{u}(t)^T\,\boldsymbol{v}(t) = 1$, and under this condition holds [4],

$$\frac{d\,\lambda(t)}{dt} = \boldsymbol{v}^T(t)\,\frac{d\,\boldsymbol{K}(t)}{dt}\,\boldsymbol{u}(t)\,. \tag{5.222}$$

For more details about the sensitivities of eigenvalues and eigenvectors see [9].