

---

# Contents

<b>1</b>	<b>What are finite elements?</b>	<b>1</b>
1.1	Introduction	1
1.2	Key points of the FE method	1
1.3	Potential energy	5
1.4	Projection	8
1.5	The error of an FE solution	12
1.6	A beautiful idea that does not work	15
1.7	Set theory	16
1.8	Principle of virtual displacements	23
1.9	Taut rope	28
1.10	Least squares	33
1.11	Distance inside = distance outside	36
1.12	Scalar product and weak solution	39
1.13	Equivalent nodal forces	41
1.14	Concentrated forces	43
1.15	Green's functions	50
1.16	Practical consequences	52
1.17	Why finite element results are wrong	55
1.18	Proof	62
1.19	Influence functions	67
1.20	Accuracy	75
1.21	Why resultant stresses are more accurate	80
1.22	Why stresses at midpoints are more accurate	84
1.23	Why stresses jump	93
1.24	Why finite element support reactions are relatively accurate	94
1.25	Gauss points	99
1.26	Local errors and pollution	105
1.27	Adaptive methods	112
1.28	St. Venant's principle	127
1.29	Singularities	129
1.30	Actio = reactio?	132
1.31	The output	135
1.32	Support conditions	137
1.33	Equilibrium	138
1.34	Changes in the temperature and displacement of supports	141

VI      Contents

1.35 Stability problems . . . . .	144
1.36 Interpolation . . . . .	148
1.37 Polynomials . . . . .	151
1.38 Infinite energy . . . . .	158
1.39 Conforming and nonconforming shape functions . . . . .	160
1.40 Partition of unity . . . . .	161
1.41 Elements . . . . .	163
1.42 Stiffness matrices . . . . .	164
1.43 Coupling degrees of freedom . . . . .	167
1.44 Numerical details . . . . .	170
1.45 Warning . . . . .	178
<b>2    What are boundary elements?</b> . . . . .	<b>181</b>
2.1 Influence functions or Betti's theorem . . . . .	182
2.2 Structural analysis with boundary elements . . . . .	189
2.3 Comparison finite elements—boundary elements . . . . .	204
<b>3    Frames</b> . . . . .	<b>211</b>
3.1 Introduction . . . . .	211
3.2 The FE approach . . . . .	212
3.3 Finite elements and the slope deflection method . . . . .	227
3.4 Stiffness matrices . . . . .	231
3.5 Approximations for stiffness matrices . . . . .	237
<b>4    Plane problems</b> . . . . .	<b>241</b>
4.1 Simple example . . . . .	241
4.2 Strains and stresses . . . . .	248
4.3 Shape functions . . . . .	251
4.4 Plane elements . . . . .	252
4.5 The patch test . . . . .	258
4.6 Volume forces . . . . .	260
4.7 Supports . . . . .	261
4.8 Nodal stresses and element stresses . . . . .	271
4.9 Truss models . . . . .	277
4.10 Two-bay wall . . . . .	278
4.11 Multistory shear wall . . . . .	280
4.12 Shear wall with suspended load . . . . .	287
4.13 Shear wall and horizontal load . . . . .	289
4.14 Equilibrium of resultant forces . . . . .	292
4.15 Adaptive mesh refinement . . . . .	296
4.16 Plane problems in soil mechanics . . . . .	300
4.17 Incompressible material . . . . .	306
4.18 Mixed methods . . . . .	307
4.19 Influence functions . . . . .	312
4.20 Error analysis . . . . .	313

4.21 Nonlinear problems .....	314
<b>5 Slabs .....</b>	<b>325</b>
5.1 Kirchhoff plates .....	326
5.2 The displacement model .....	331
5.3 Elements .....	332
5.4 Hybrid elements .....	335
5.5 Singularities of a Kirchhoff plate .....	339
5.6 Reissner–Mindlin plates .....	341
5.7 Singularities of a Reissner–Mindlin plate .....	346
5.8 Reissner–Mindlin elements .....	349
5.9 Supports .....	351
5.10 Columns .....	353
5.11 Shear forces .....	361
5.12 Variable thickness .....	362
5.13 Beam models .....	364
5.14 Wheel loads .....	369
5.15 Circular slabs .....	369
5.16 T beams .....	372
5.17 Foundation slabs .....	378
5.18 Direct design method .....	384
5.19 Point supports .....	386
5.20 Study .....	386
<b>6 Shells .....</b>	<b>391</b>
6.1 Shell equations .....	391
6.2 Shells of revolution .....	394
6.3 Volume elements and degenerate shell elements .....	396
6.4 Circular arches .....	397
6.5 Flat elements .....	399
6.6 Membranes .....	404
<b>7 Theoretical details .....</b>	<b>409</b>
7.1 Scalar product .....	409
7.2 Green's identities .....	414
7.3 Green's functions .....	419
7.4 Generalized Green's functions .....	421
7.5 Nonlinear problems .....	428
7.6 The derivation of influence functions .....	432
7.7 Shifted Green's functions .....	437
7.8 The dual space .....	447
7.9 Some concepts of error analysis .....	453
7.10 Important equations and inequalities .....	461
<b>References .....</b>	<b>471</b>